Empirically Comparing the Finite-Time Performance of Simulation-Optimization Algorithms

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Simulation Optimization (SO)

Optimize a real-valued objective function, estimated via simulation, over a deterministic domain.

Challenges:

1. Error in estimating objective function.
2. Unknown topology (continuity/differentiability/convexity).

SO algorithms are designed to solve a broad class of problems.

SO Algorithm Performance

Many theoretical results for asymptotic performance,

- . . . as simulation budget approaches infinity.

Examples:

- Algorithm converges to local (global) optimizer.
- Convergence rate, once within neighborhood of optimizer.

Required budget for such results may exceed practical budget!

To decide which algorithm to use, a practitioner cares about **finite-time** performance.

Evaluating Algorithms

SO community lags behind other optimization communities:

- Established testbed of problems for benchmarking.
- Metrics for empirical finite-time performance.
- Comparison of algorithms on large testbed.

We implement several popular SO algorithms and test them on a subset of problems from the SimOpt library (`www.simopt.org`).

Objectives

Near-term:

1. Comparison of finite-time performance of different algorithms.
2. Insights on the types of problems on which certain algorithms work well.

Long-term:

1. More contributions to SimOpt library.
   - Problems and/or algorithms.
2. Development of finite-time performance metrics.
   - E.g., adapting performance profiles.
3. Motivate others to do similar comparisons.
4. Development of algorithms with strong finite-time performance.

Evaluating Finite-Time Performance

In deterministic optimization:

- Measure computational effort needed to get to optimal solution (or within specified tolerance).
  - Number of function evaluations or wall clock time.

Doesn't work so well for SO.

- Optimal solution is often unknown.
- Often no certificate of optimality.
- Estimation error makes it hard to check tolerance condition.

Evaluating Finite-Time Performance

Idea

1. Fix a simulation budget.
2. Evaluate the objective function at the estimated best solution found within the budget.

We measure the budget in number of objective function evaluations.

Evaluating Finite-Time Performance

Let $Z(n)$ be the **true** objective function value of the **estimated** best solution visited in the first n objective function evaluations.

- $Z(n)$ is a random variable, because the estimated best solution, $X(n)$, is random.
- Conditional on $X(n)$, $Z(n)$ is fixed, but needs to be estimated.

In our experiments, we estimate $Z(n)$ (conditional on $X(n)$) in a *post-processing* step.

Evaluating Finite-Time Performance

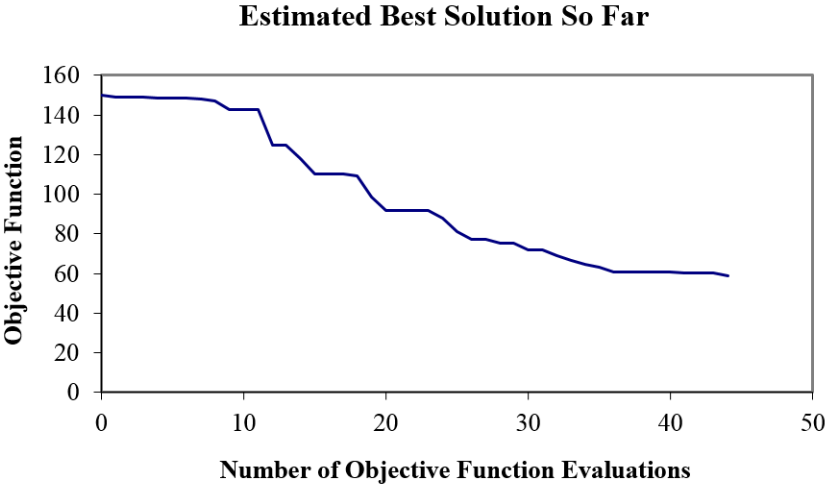


Figure: Pasupathy and Henderson (2006).

Evaluating Finite-Time Performance

Can obtain $Z(n)$ curve from single macroreplication of an algorithm.

- Unless the algorithm uses the budget in setting parameters.

Location of $Z(n)$ curve is random, so take several macroreplications and look at:

- mean,
- median/quantile,
- empirical cdf *(hard to show on one plot)*.

Algorithms

1. Random Search (non-adaptive)
2. Gradient Search with Random Restarts
   - Central finite differences for gradient estimate.
3. Simultaneous Perturbation Stochastic Approximation (SPSA)
   - Uses budget to set gain sequence.
4. Stochastic Trust-Region Response-Surface Method (STRONG)
   - Didn't use design of experiments for fitting.
   - Central finite differences for gradient estimate.
     a. BFGS estimate of Hessian. (STRONG)
     b. No second-order model. (STRONG-Stage1)
5. Nelder-Mead
   - Simplicial method that doesn't use gradient information.

Benchmark Problems

Table: SimOpt benchmark problems and their characteristics.

Name on SimOpt	Dimension	Optimal Solution
A Multimodal Function	2	Known
Ambulances in a Square	6	Unknown
Continuous Newsvendor	1	Known
Dual Sourcing	2	Unknown
Economic-Order-Quantity	1	Known
Facility Location	4	Unknown
GI/G/1 Queue	1	Unknown
M/M/1 Metamodel	3	Known
Optimal Controller for a POMDP	10	Unknown
Optimization of a Production Line	3	Unknown
Parameter Estimation: 2D Gamma	2	Known
Rosenbrock's Function	40	Known
Route Prices for Mobility-on-Demand	12	Unknown
SAN Duration	13	Unknown
Toll Road Improvements	12	Unknown

Problems

Properties of all problems:

- Continuous decision variables.
- Deterministic (box) constraints or unbounded.

Initial solution is drawn from probability distribution over domain.

- Uniform distribution for bounded variables.
- Exponential/Laplace distribution for unbounded variables.

Took 30 replications at given solution to estimate its objective value.

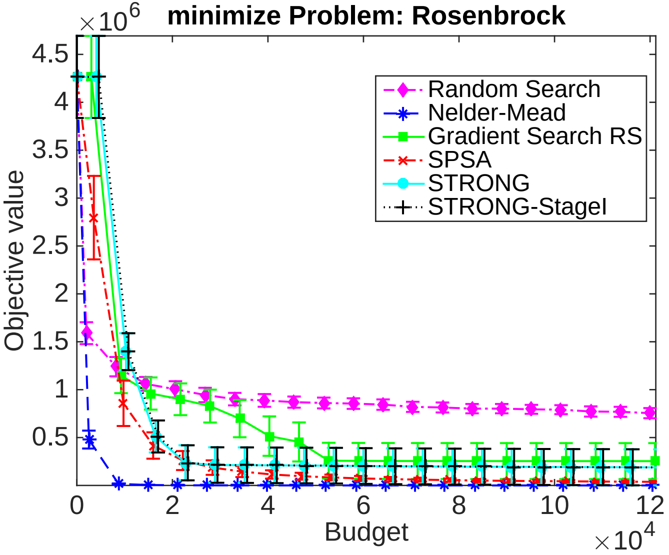
- Used common random numbers (CRN) across solutions.

Experiments

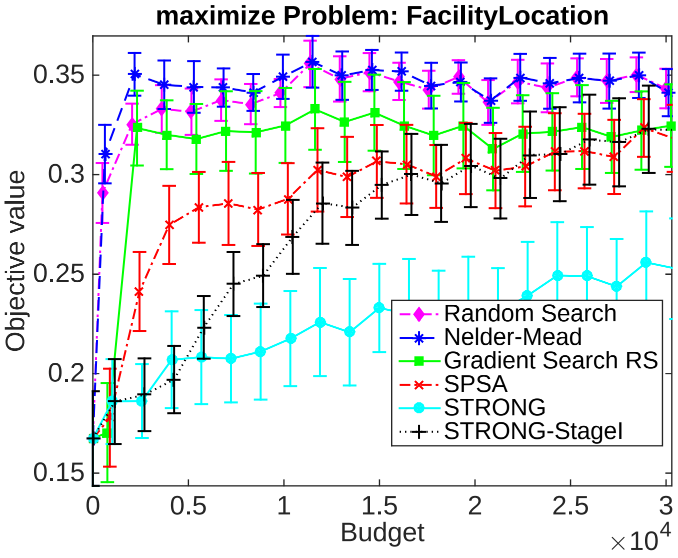
For every problem:

- Ran 30 macroreplications of each algorithm.
- Recorded estimated best solution $X(n)$ for range of n values.
- Ran 30 function evaluations at each $X(n)$ to estimate $Z(n)$ conditional on $X(n)$.
- Averaged 30 estimates of $Z(n)$.
- Constructed 95% normal confidence intervals.

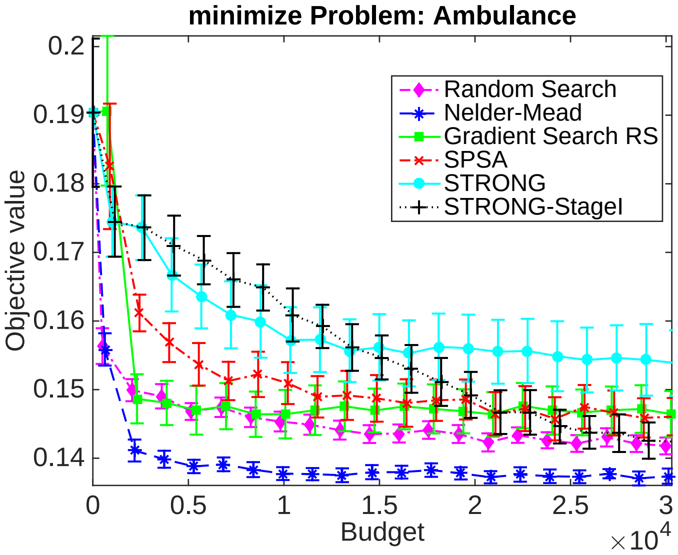
Rosenbrock ($dim = 40$)



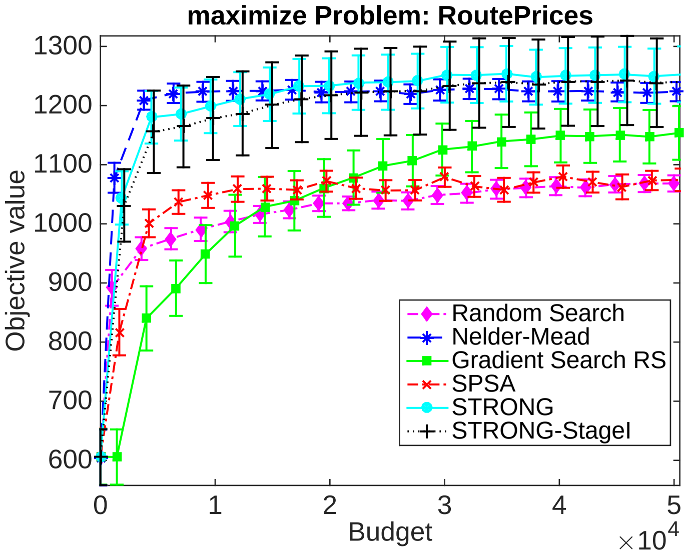
Facility Location ($dim = 4$)



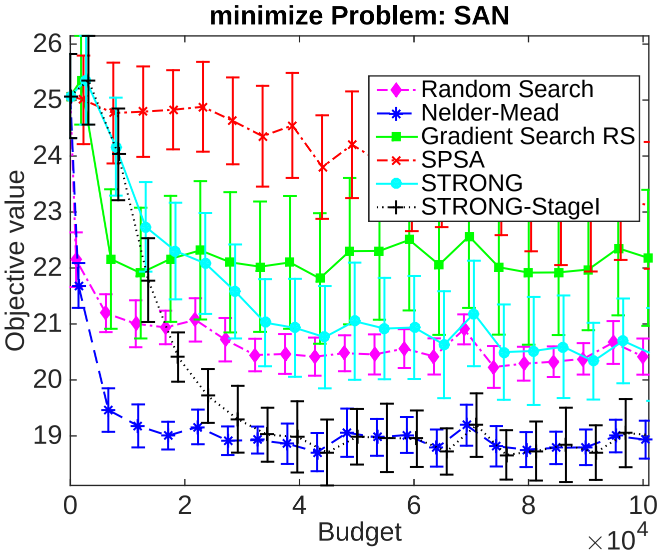
Ambulance ($dim = 6$)



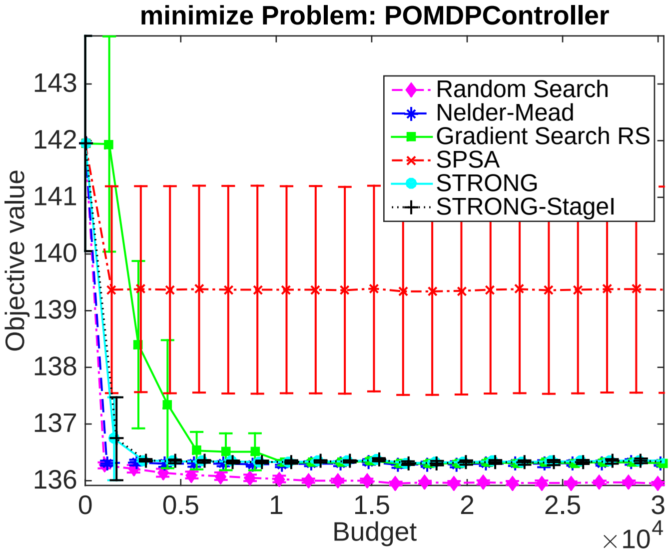
Route Prices ($dim = 12$)



Stochastic Activity Network ($dim = 13$)



POMDP Controller ($dim = 10$)



Conclusions

Takeaways

1. Robust performance of Nelder-Mead across problems.
2. STRONG-Stage1 did as well as (or better than) STRONG.
3. Random Search did better than expected.
4. Performance of SPSA was sometimes highly variable.

Future Work

Similar comparisons for:

- High-dimensional problems.
- Discrete/integer-ordered variables.
- Stochastic constraints.

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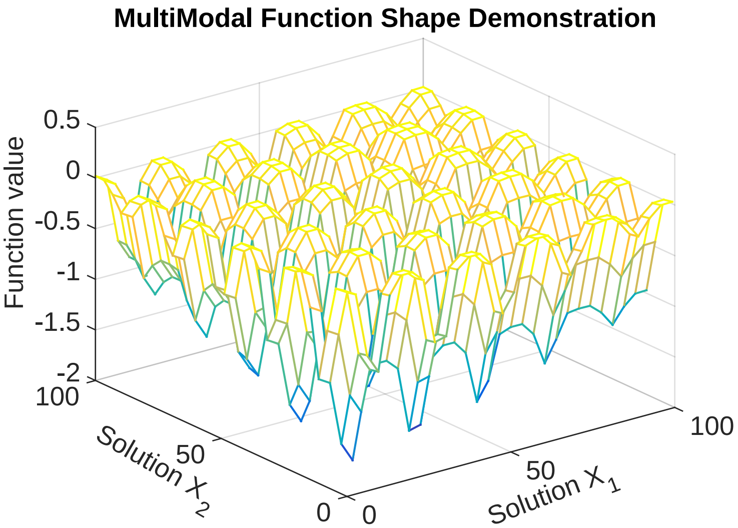
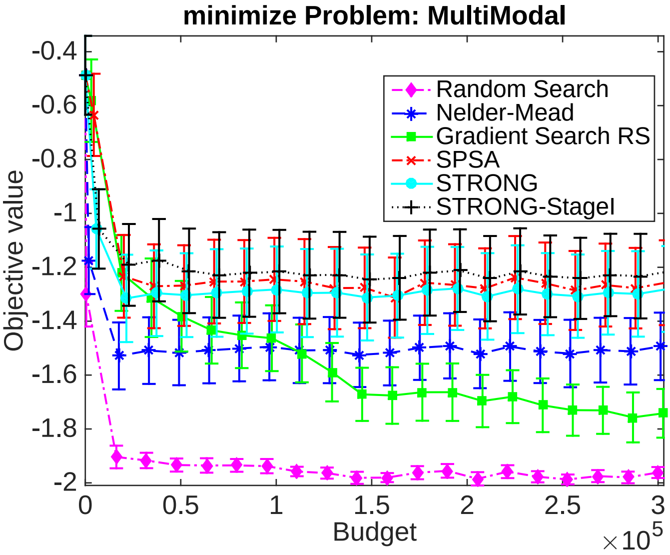
MultiModal ($dim = 2$)



MultiModal ($dim = 2$)



Ambulance ($dim = 6$)