Challenges in Applying Ranking and Selection after Search

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Motivation

Setting: Large-scale simulation optimization

- Optimize a noisy function over a large number of systems.
- Simulation budget only allows for evaluating a subset of candidate systems.
- Ultimately choose a system as the best.

Goal: A finite-time statistical guarantee on the quality of the chosen system *relative* to the other candidate systems.

Method:

1. Identify candidate systems via sampling or search.
2. Run a ranking-and-selection (R&S) procedure on the candidate systems.

Research Question and Results

Q: *Do the guarantees of R&S procedures hold when replications taken during search are reused?*

A: No. Not in general, though conservative procedures are likely robust. This finding extends to selection procedures for non-normal data, e.g. multi-armed bandits in full-exploration setting.

Existing R&S procedures that reuse search data:

- Boesel et al. [2003]
- Pichitlamken and Nelson [2006]
- Hong and Nelson [2007]

What's the Problem with Search Data?

Observation:
The *identities* of the returned systems depend on the *observed performance* of previously visited systems.

$\Downarrow$

Search replications are conditionally dependent given the sequence of returned systems.

We design a "search-like" method that exploits this dependence to weaken R&S guarantees.

Adversarial Search (AS):

- If best system looks best $\rightarrow$ add a δ-better system.
- If best system doesn't look best $\rightarrow$ add a δ-worse system.

Traditional R&S Procedures

Assumptions:

- Fixed set of k systems with unknown performance.
- Replications are i.i.d. normal, independent across systems.

Formulations:

- Selection: select one system.
- Subset-Selection: preserve a subset of systems.

Events:

- Correct Selection (CS): select (or preserve) the best system.
- Good Selection (GS): select (or preserve) a system strictly within δ of the best.

Zones:

- Preference Zone (PZ(δ)): the best system is at least δ better than all the others.
- Indifference Zone (IZ(δ)): the complement of PZ(δ).

Guarantees:

$$\mathbb{P}(\text{CS}) \geq 1-\alpha \quad \forall \mu \in \text{PZ}(\delta), \quad (\text{PCS})$$
$$\mathbb{P}(\text{GS}) \geq 1-\alpha \quad \forall \mu, \quad (\text{PGS})$$

for $1/k < 1-\alpha < 1$ and $\delta > 0$ where μ is the configuration of the true means of the systems.

R&S after Search

Apply a R&S procedure on a set of systems $\mathcal{X}$ determined by a sampling or search method $\mathcal{S}$.

What are meaningful PCS/PGS guarantees?

Guarantees Conditional on $\mathcal{X}$:

$$\mathbb{P}(\text{CS after } \mathcal{S} \mid \mathcal{X}) \geq 1-\alpha \quad \forall \mathcal{X} \text{ s.t. } \mu(\mathcal{X}) \in \text{PZ}(\delta),$$
$$\mathbb{P}(\text{GS after } \mathcal{S} \mid \mathcal{X}) \geq 1-\alpha \quad \forall \mathcal{X}.$$

$\Downarrow$

Overall Guarantees:

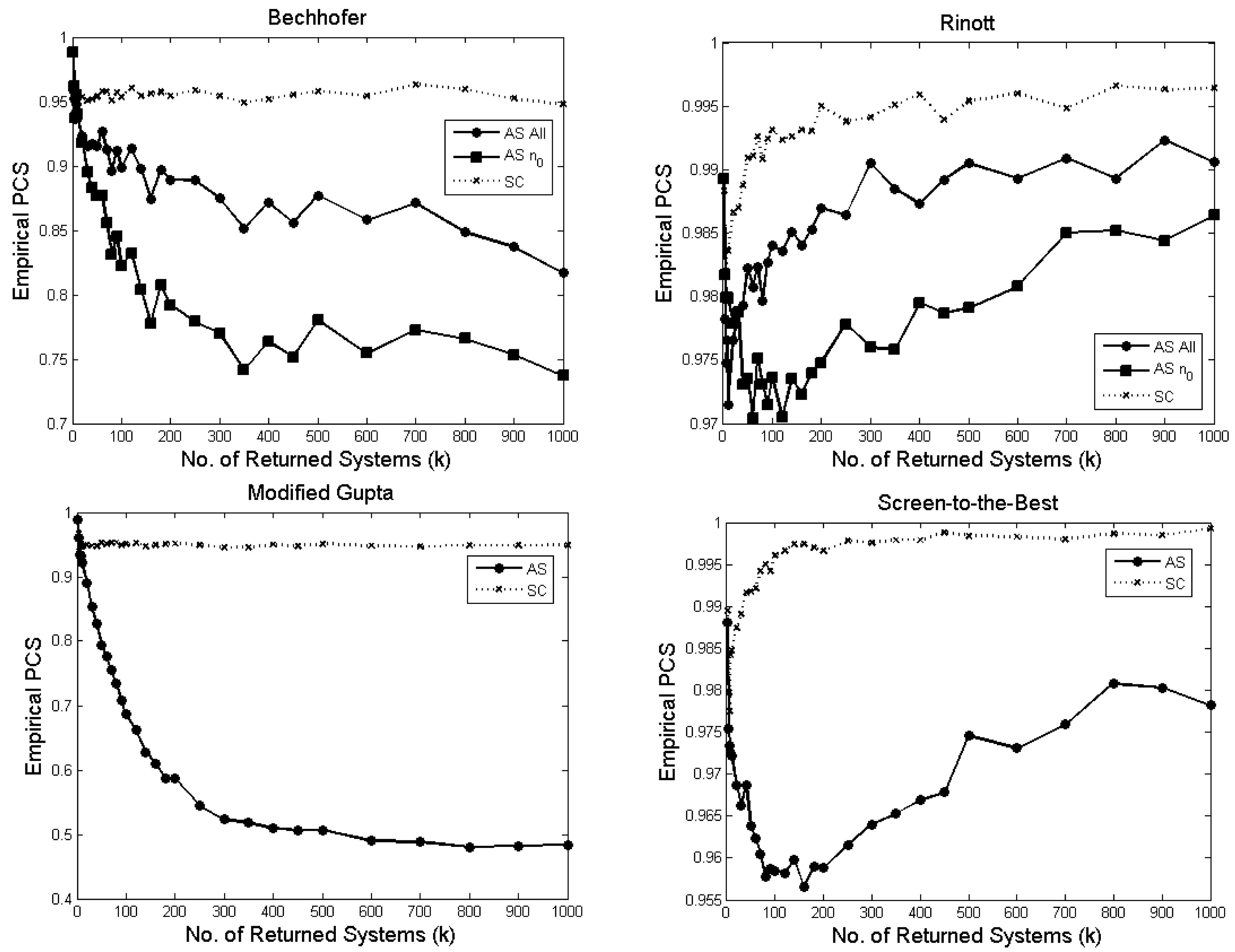
$$\mathbb{P}(\text{CS after } \mathcal{S} \mid \mu(\mathcal{X}) \in \text{PZ}(\delta)) \geq 1-\alpha,$$
$$\mathbb{P}(\text{GS after } \mathcal{S}) \geq 1-\alpha.$$

Guarantees that require $\mu(\mathcal{X}) \in \text{PZ}(\delta)$ are ill-suited to the framework of R&S after search.

- Returned systems will likely have similar performance as a search progresses.
- No control over whether $\mu(\mathcal{X}) \in \text{PZ}(\delta)$.

Simulation Experiments with AS

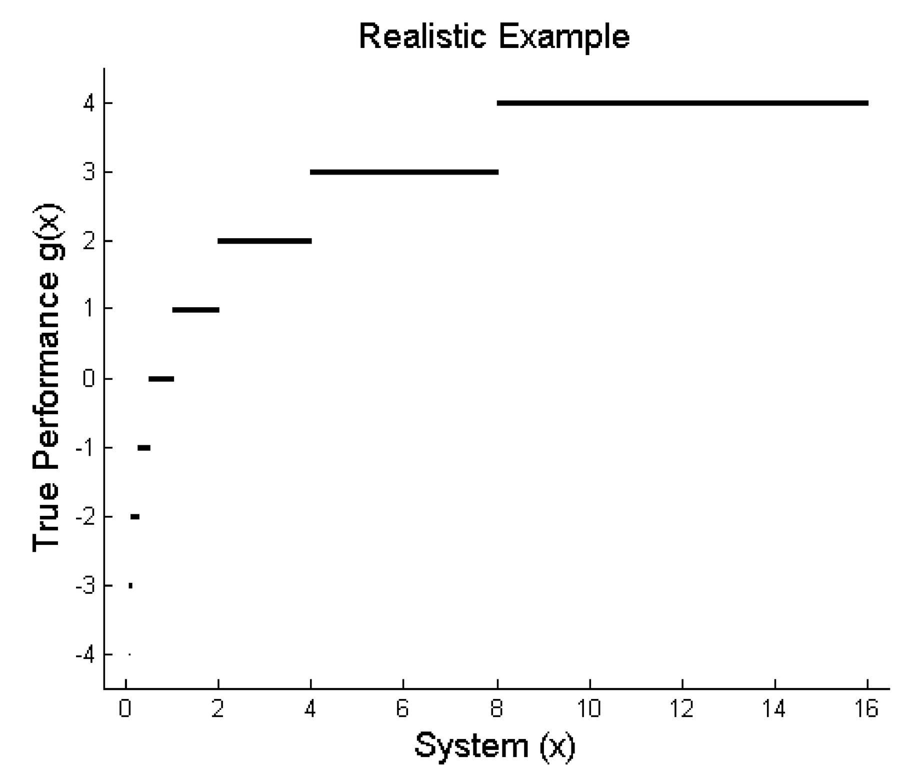
Plotted overall PCS for two selection procedures (Bechhofer and Rinott) and two subset-selection procedures (Modified Gupta and Screen-to-the-Best).

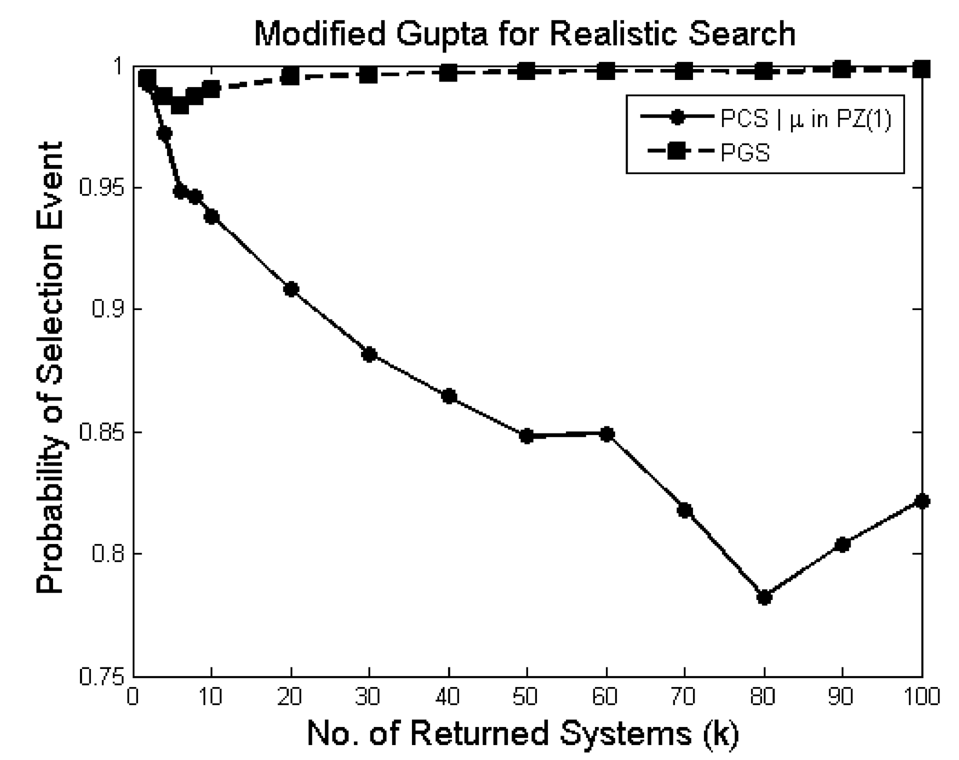


Realistic Search

Maximize $\lceil \log_2 x \rceil$ on the interval $[1/16, 16]$.

- Start at $x_1 = 0.75$ and take $n_0 = 10$ replications.
- Choose next system uniformly from within ± 1 of best-looking system.





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