Optimal Pinging Frequencies in the Search for an Immobile Beacon

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Motivation

Deep sea searches for missing aircraft

- Air France Flight 447 (2009)
- Malaysia Airlines Flight 370 (2014)

Flight data recorder (FDR)

- AKA “black box”
- Keeps electronic record of aircraft operations
- **Extremely** important to follow-up investigations

Finding the Flight Data Recorder

Each FDR is equipped with an underwater locator beacon (ULB).

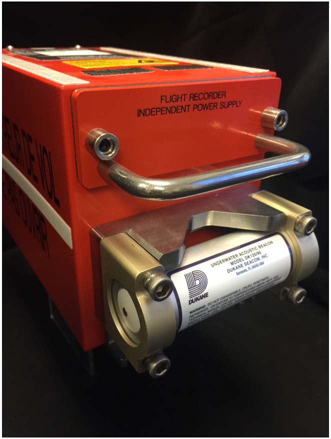

Figure: Flight recorder (orange box) with ULB (silver cannister).

Underwater Locator Beacon

How the ULB works:

- Activates when submerged in water
- Produces ultrasonic pings (roughly once per second)
- Battery life of $\approx$30 days once activated

Finding the FDR before the ULB's battery dies is **critical**

- Other search methods are less effective and/or slower
- E.g., high altitude fly-overs, side-scan sonar searches

Finding the Underwater Locator Beacon

Ocean-surface search vessels

- Pass over search area on parallel runs
- Drag a towed pinger locator (TPL) ≈1000 ft above ocean floor
- Move slowly (2-3 mph) and turn slowly (3-8 hrs)

Figure: Search path over a rectangular section of the search area.

Search with Towed Pinger Locator

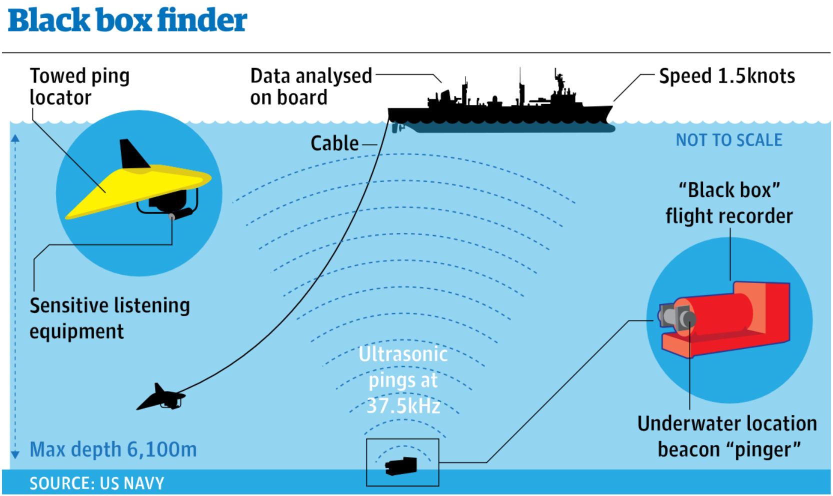

Figure: Ocean-surface search using towed pinger locator.

After Detection

Once pings are detected

- Perpendicular runs to box in FDR position
- Triangulation
- Recovery by diver, submersible, or remote-operated vehicle

Beacon Design

Proposed recommendations by governing bodies:

1. Add a second beacon with
   - lower frequency of sound
   - increased range of detection (8 miles vs 2.5 miles)
2. Extend battery life from 30 days to 90 days

A less costly alternative:

Modify the beacon's pinging period—time between successive pings

Beacon Design Question

Need to determine the pinging period before other search parameters (e.g., search speed) are known.

Is the industry-standard pinging period...

- ...too short?
- ...too long?
- ...just right?

Method: develop a simplified search model to get a first-order answer.

Existing Search Models

We adapt the linear search problem (LSP) for an immobile object.

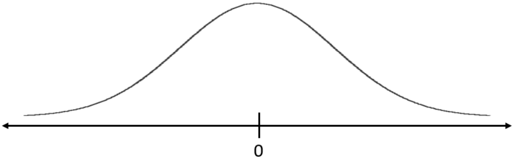

Figure: LSP with distribution of hidden object.

Novelties

1. Object is intermittently detectable
2. No switching/turning
3. Search vessel speed is selected from known distribution

Our Model

Assumptions

- Linear search problem (by unfolding parallel runs)
- Definite range law
  - $P(\text{Detect ping}) = 1$ if within range during ping
- A single search vessel
- The search terminates once a ping is detected

Notation

- $[0, L]$: search space represented as an interval
- B: location of beacon
- r: radius of detection
- $n + 1$: number of pings (including at time 0)
- ν: search speed
- τ: pinging period

Fixed Search Speed ν

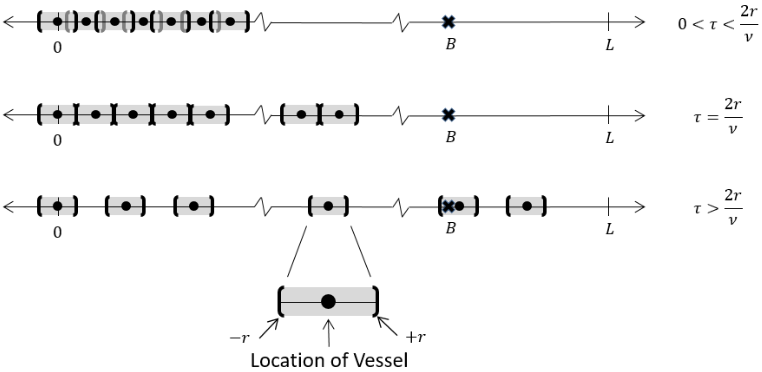

Figure: Search on $[0, L]$ under three settings of the pinging period τ.

Probability of Detection

Assume B uniformly distributed on $[0, L]$.

Let $\theta(\nu, \tau)$ denote the probability of detection for a search speed ν and pinging period τ.

Objective

Find τ^* that maximizes $\theta(\nu, \tau)$, assuming ν is fixed and known.

Two Cases

1. **Sufficient pings:** $n \geq \frac{L-r}{2r}$
   - Enough pinging instances to search entire interval $[0, L]$.
2. **Insufficient pings:** $n < \frac{L-r}{2r}$

Sufficient Pings

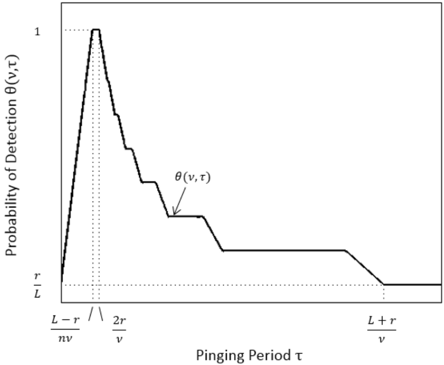

Figure: The probability of detection as a function of the pinging period.

$$\tau^* = \left[\frac{L-r}{n\nu}, \frac{2r}{\nu}\right]$$

Insufficient Pings

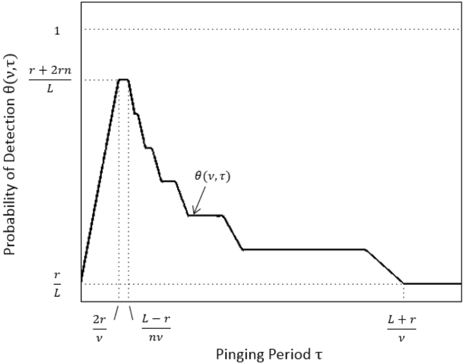

Figure: The probability of detection as a function of the pinging period.

$$\tau^* = \left[\frac{2r}{\nu}, \frac{L-r}{n\nu}\right]$$

Two Search Speeds

Suppose search speed has distribution

$$\nu = \begin{cases} \nu_1 & \text{w.p. } \lambda \\ \nu_2 & \text{w.p. } 1-\lambda \end{cases}$$

for $\lambda \in (0,1)$ and $\nu_1 < \nu_2$.

$$E_\nu[\theta(\nu,\tau)] = \lambda\theta(\nu_1,\tau) + (1-\lambda)\theta(\nu_2,\tau).$$

Objective

Find τ^* that maximizes $E_\nu[\theta(\nu,\tau)]$, assuming ν_1, ν_2, and λ are fixed and known.

Sufficient Pings

Assume $\nu_1 < \frac{L-r}{2rn}\nu_2$, otherwise $\tau^* = 2r/\nu$ is optimal.

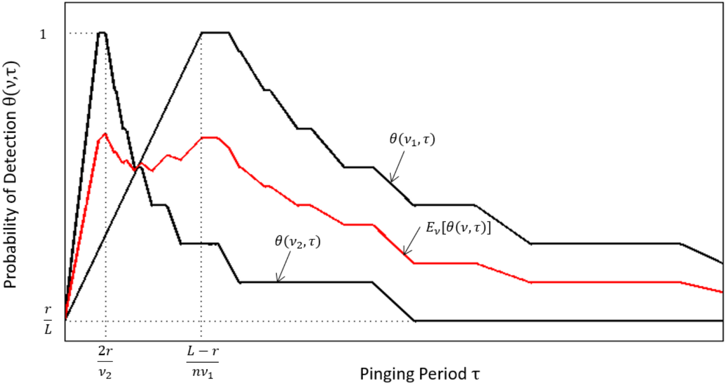

Figure: Expected probability of detection (red line) as a function of the pinging period for two speeds with $\lambda = 1/2$. From picture, $\tau^* \in \left[\frac{2r}{\nu_2}, \frac{L-r}{n\nu_1}\right]$.

Solution to 2-Speed Problem

Theorem

There exists a threshold $\tilde{\lambda}$ such that

$$\tau^* = \begin{cases} \frac{2r}{\nu_2} & \text{(left endpoint)} \quad \text{for } \lambda \leq \tilde{\lambda}, \\ \frac{L-r}{n\nu_1} & \text{(right endpoint)} \quad \text{for } \lambda \geq \tilde{\lambda}. \end{cases}$$

An explicit expression for $\tilde{\lambda}$ in terms of the search speeds ν_1 and ν_2 can be easily solved.

Sketch Proof

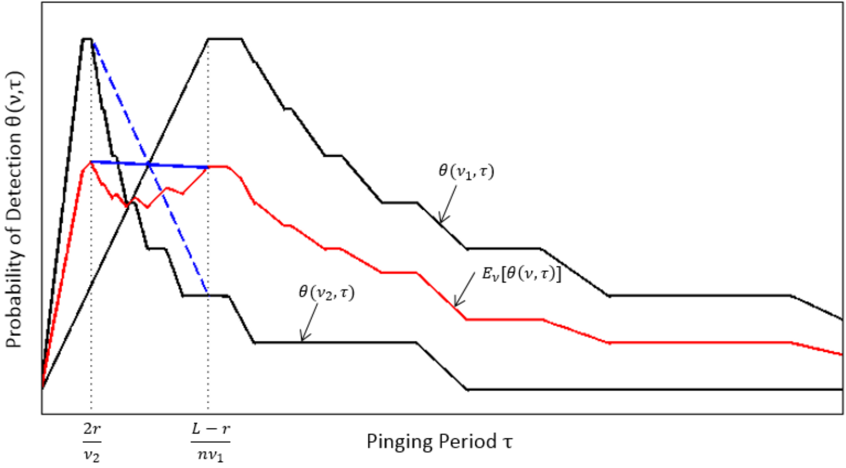

Figure: Linear upper bounds on probability of detection (dashed blue line) and expected probability of detection (solid blue line). Tight at endpoints.

More about $\tilde{\lambda}$

Corollary

When the faster search speed ν_2 is as or more likely than the slower search speed ν_1, the optimal pinging period is the longest period that ensures no intervals are left undetected between pings; i.e., $\tau^* = 2r/\nu_2$.

That is, $\tilde{\lambda} \geq 1/2$.

Insufficient Pings

Harder case because the linear upper bound may not exist.

Proposition

If the faster search speed ν_2 is more likely than the slower search speed ν_1, i.e., $\lambda < 1/2$, then $\tau^* = \frac{L-r}{n\nu_2}$.

Three or more search speeds

Closed-form solutions are harder to come by, but using a one-dimensional optimization algorithm is always an option.

Case Study: MH370

- Disappeared over Indian Ocean
- Preliminary air and sea searches found no matching debris
- Revised search area of 85,000 sq-miles wherein ocean depth is between 10,000–15,000 ft

Figure: Search area for MH370.

Case Study: MH370

Conservative estimates for variables

- Shortest range of detection: $r = 0.6$ miles
- Fastest search speed: $\nu = 5.8$ mph

Calculation

$$\tau^* = \frac{2r}{\nu} = \frac{2(0.6 \text{ miles})}{5.8 \text{ mph}} = 12.4 \text{ minutes.}$$

Industry standard pinging period: $\tau = 1.1$ seconds.

Difference in magnitude of ≈ 700!

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Sources

Images

- Black Box and ULB: `http://www.radiantpowercorp.com/wp-content/uploads/2015/07/mounted_beacon.jpg`
- TPL: `https://static.guim.co.uk/ni/1395851242682/Black-box-finder_WEB.svg`
- MH370 Search Area: `https://www.extremetech.com/wp-content/uploads/2016/07/SearchArea.png`

References

- Optimal pinging frequencies in the search for an immobile beacon. Eckman, D., Maillart, L., and Schaefer, A. *IIE Transactions*, 48(6): 489–500. 2016.